

Magnetic field generation in Higgs inflation model

Moumita Das^{1,*} and Subhendra Mohanty^{1,†}

¹*Physical Research Laboratory, Ahmedabad 380009, India*

Abstract

We study the generation of magnetic field in Higgs-inflation models where the Standard Model Higgs boson has a large coupling to the Ricci scalar. We couple the Higgs field to the Electromagnetic fields via a non- renormalizable dimension six operator suppressed by the Planck scale in the Jordan frame. We show that during Higgs inflation magnetic fields with present value 10^{-6} Gauss and comoving coherence length of $100kpc$ can be generated in the Einstein frame. The problem of large back-reaction which is generic in the usual inflation models of magneto-genesis is avoided as the back-reaction is suppressed by the large Higgs-curvature coupling.

*Electronic address: moumita@prl.res.in

†Electronic address: mohanty@prl.res.in

1. INTRODUCTION

Observations [1, 2] of magnetic field associated with high red-shift ($z > 1$) galaxies suggest that the large scale magnetic fields have a cosmological rather than an astrophysical origin. In the dynamo theory of magnetic field amplification in galaxies [3, 4] and initial seed B- field of strength 10^{-20} G can be amplified to the observed 10^{-6} G by the magnetohydrodynamics of galactic rotation. However observations [1, 2] show that the magnetic fields associated with galaxies have a very narrow spread around a micro-Gauss and therefore independent of the number of rotations of the galaxies. In the standard hot big bang model of cosmology the generation of magnetic fields of large coherence scales (1kpc - 1 Mpc) runs into problems with causality. For example if B-fields are generated in the electro-weak era then the present coherence length of the 100 kpc would correspond to a length scale $\lambda_{EW} = 100kpc(T_0/100GeV) = 10^7cm$. This length scale is much larger than the distance scale of causal Horizon at the electro-weak era $H_{EW}^{-1} = 10^{-2}cm$. This suggests that if B-fields have an origin in the fundamental interaction then the perturbations must be super-horizon which can happen during inflation. The magnetic field generated with the coherence scale H_I during the time of inflation can be as large as the present horizon H_0 . Generation of magnetic field during inflation has been studied extensively [4–6] starting with Turner and Widrow [7] who coupled electromagnetic fields with curvature and the axion-inflaton and by Ratra [8] who coupled electromagnetic field with the dilaton-inflaton. However in recent studies [9–12] it has been observed that in theories where B-field is generated during the inflation the fluctuations of the electromagnetic field are as large as the perturbations of the inflaton and spoil the prediction of near-scale invariant primordial density perturbation of inflation.

A model of inflation with the standard model Higgs field has been proposed [13, 14] in which the Higgs has a large coupling with the Ricci scalar, $\xi\phi^2R$ in the Jordan frame. This has the interesting property that the Higgs potential $V = \lambda\phi^4$ in the Jordan frame transforms to a flat potential $\hat{V} \simeq \lambda M_P^4/(4\xi^2)$ in the Einstein frame in the early universe (when $\phi > M_P/\xi$) which gives almost scale invariant density perturbations $\Delta_R \sim 10^{-5}$ with the parameters chosen as $\lambda = 1$ and $\xi = 4.6 \times 10^4$. In the present epoch where $\phi = v = 246$ GeV, the Ricci coupling term of the Higgs is negligibly small compared to the standard M_P^2R term of gravity and the Higgs potential in both Einstein and Jordan frames is $\lambda(\phi^2 - v^2)^2$

leading to the standard Higgs mass $m_h = \sqrt{2\lambda}v$ with $\lambda \sim 1$ (instead of $\lambda \sim 10^{-13}$ as would be required to get the inflationary curvature perturbations of the right amplitude in an inflaton model with a $\lambda\phi^4$ potential without the added curvature coupling). The Higgs mass $\sqrt{2\lambda}v$ at scales M_P/ξ must be renormalised down to the electroweak scale which leads to predictions for the Higgs mass to be in the range (126 GeV-195 GeV) [15, 16].

In this paper we introduce a non-renormalisable coupling of the Higgs with the electromagnetic fields of the form $\frac{\phi^\dagger\phi}{M_P^2}F^2$. This is the leading order term in inverse powers of some large mass scale (which we take to be M_P) which is invariant under the standard model symmetry group. This term breaks the conformal symmetry and generates a magnetic field at the time of inflation when $\phi \sim M_P/\xi$. We find that the magnetic field generated at the time of inflation in terms of the Hubble parameter \hat{H} is $\delta_B \sim \hat{H}^2\xi$. This can be compared with fluctuation of the electromagnetic energy density $\rho_{EM} = (\phi^2/M_P^2)\delta_B^2 \sim \hat{H}^4$. The main point we illustrate in this paper is that if one starts with a conformal symmetry breaking electromagnetic interaction in the Jordan frame then one can generate a larger magnetic field by a factor of $\sqrt{\xi}$ in the Einstein frame and in addition the electromagnetic backreaction is smaller compared to the magnetic field. We show that with $\hat{H} \sim 10^{13}$ GeV and $\xi = 4.6 \times 10^4$ as required by the observations of the primordial density perturbations, one can generate magnetic fields of order 10^{-6} Gauss in the present universe with coherence length of 100 kpc.

2. HIGGS-INFLATION

Consider the Higgs scalar $\Phi = \frac{1}{\sqrt{2}}(0, v + \phi)^T$ (in the unitary gauge) with a non-minimal coupling to the Ricci scalar, a Higgs potential $V(\phi)$ and a Higgs-photon interaction,

$$S_J = \int d^4x \sqrt{-g} \left[-\frac{M_p^2 + \xi\phi^2}{2}R + \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi) - \frac{1}{4}I^2(\phi)F^{\mu\nu}F_{\mu\nu} \right] \quad (1)$$

where $V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$ Higgs potential with $v = 246$ GeV and I is the inverse of the electromagnetic coupling $I = 1/g$. Here we assume $I(\phi)$ has the explicit dependence on ϕ as,

$$I^2(\phi) = \frac{\phi^\dagger\phi}{M_p^2} \quad (2)$$

The metric in this ‘Jordan frame’ (the frame in which there is a non-minimal Ricci coupling with the Higgs) is assumed to be,

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \quad (3)$$

The non-minimal coupling with the gravity can be removed by making a conformal transformation to the ‘Einstein frame’ [17–19],

$$g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \text{where} \quad \Omega^2 = 1 + \frac{\xi\phi^2}{M_p^2} \quad (4)$$

In Einstein frame, the action will look like,

$$S_E = \int d^4x \sqrt{-\hat{g}} \left[-\frac{M_p^2}{2} \hat{R} + \frac{1}{2\Omega^2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} + \frac{3\xi^2}{M_p^2 \Omega^4} (\phi \phi_{,\mu})^2 - \frac{V(\phi)}{\Omega^4} - \frac{1}{4} I^2(\phi) F^{\mu\nu} F_{\mu\nu} \right] \quad (5)$$

This conformal transformation produces non-canonical kinetic terms for the scalar field ϕ . To make the kinetic term of the ϕ field canonical, we have to redefine the ϕ field in terms of new scalar,

$$\frac{d\hat{\phi}}{d\phi} = \sqrt{\frac{\Omega^2 + \frac{6\xi^2\phi^2}{M_p^2}}{\Omega^4}} \quad (6)$$

The the action (5), in terms of $\hat{\phi}$ is ,

$$S_E = \int d^4x \sqrt{-\hat{g}} \left[-\frac{M_p^2}{2} \hat{R} + \frac{1}{2} \hat{g}^{\mu\nu} \hat{\phi}_{,\mu} \hat{\phi}_{,\nu} - \frac{V(\hat{\phi})}{\Omega^4} - \frac{1}{4} I^2(\hat{\phi}) F^{\mu\nu} F_{\mu\nu} \right] \quad (7)$$

where ϕ is an implicit function of $\hat{\phi}$.

When $\phi \ll M_P/\xi$, $\Omega \simeq 1$ and $\hat{\phi} = \phi$. This corresponds to the situation in the present era where $\phi = v$. Inflation takes place when $\phi \gg M_P/\xi$ and in this regime $\Omega \simeq \sqrt{\xi}\phi/M_P$ and the relation between ϕ and $\hat{\phi}$ obtained from (6) is,

$$\phi = \frac{M_p}{\sqrt{\xi}} \exp\left(\frac{\hat{\phi}}{\sqrt{6}M_p}\right) \quad (8)$$

And the Higgs potential in this limit will be,

$$\hat{V} = \frac{V}{\Omega^4} \simeq \frac{\lambda\phi^4}{4\Omega^4} = \frac{\lambda M_p^4}{4\xi^2} \left(1 + \frac{M_p^2}{\xi\phi^2}\right)^{-2} \quad (9)$$

Now using equation (8), we can write \hat{V} in terms of $\hat{\phi}$ as follows,

$$\hat{V} = \frac{\lambda M_p^4}{4\xi^2} \left(1 + \exp\left(-\frac{2\hat{\phi}}{\sqrt{6}M_p}\right)\right)^{-2} \quad (10)$$

Since, $\phi > \frac{M_p}{\sqrt{\xi}}$, the Higgs-inflaton has the exponentially flat potential in the Einstein frame.

Due to the conformal transformation, the metric becomes,

$$d\hat{s}^2 = \Omega^2 ds^2 = d\hat{t}^2 - \hat{a}^2(\hat{t})\delta_{ij}dx^i dx^j \quad (11)$$

where $\hat{a} = \Omega a$ and $d\hat{t} = \Omega dt$.

The inflaton field satisfies the following equation,

$$\frac{d\hat{\phi}}{d\hat{t}} = -\frac{\hat{V}'}{3\hat{H}} \quad (12)$$

where prime denote the derivative with respect to $\hat{\phi}$. Solving this equation, we can find the relation between ϕ and scale factor \hat{a} ,

$$\phi = \frac{2M_p}{\sqrt{3\xi}}(\log \hat{a})^{1/2} \quad (13)$$

Using the equation (2) , we can write $I(\phi)$ as a function of ' \hat{a} ' as follows,

$$I(\phi) = \frac{2}{\sqrt{3\xi}}(\log \hat{a})^{1/2} \quad (14)$$

For the calculation of δ_B , we have to know the value of ξ , which can be calculated from the curvature perturbation. The amplitude of the curvature perturbation can be written as,

$$\Delta_R^2 = \frac{1}{4\pi^2} \left(\frac{\hat{H}^2}{d\hat{\phi}/d\hat{t}} \right)^2 = \frac{3}{8\pi^2} \frac{\hat{H}^4}{\epsilon} \quad (15)$$

where, $H^2 = \frac{8\pi G}{3}\hat{V} = \frac{\hat{V}}{3M_p^2}$, $d\hat{\phi}/d\hat{t} = -\hat{V}'/3\hat{H}$ and $\epsilon = \frac{M_p^2}{2} \left(\frac{\hat{V}'}{\hat{V}} \right)^2 \simeq \frac{4M_p^4}{3\xi^2\phi^4}$. Therefore, the amplitude of the curvature perturbation becomes,

$$\Delta_R^2 = \frac{1}{24\pi^2} \frac{\hat{V}}{\epsilon} \frac{1}{M_p^4} = 5.23 \frac{\lambda}{\xi^2} \quad (16)$$

where we have taken $\hat{V} \simeq \frac{\lambda M_p^4}{4\xi^2}$ and $\phi = 9.01 \frac{M_p}{\sqrt{\xi}}$ for $N = 61$. Taking $\lambda = 1$ and the WMAP result [20] $\Delta_R^2 = 2.43 \times 10^{-9}$, we see that $\xi = 4.6 \times 10^4$. One can check the value of ξ which gives the correct amplitude of curvature perturbation predicts that the spectral index n_s is,

$$n_s = 1 + 2\eta - 6\epsilon = 0.965 \quad (17)$$

(where $\eta = M_p^2 \left(\frac{\hat{V}''}{\hat{V}} \right) \simeq -\frac{4M_p^2}{3\xi\phi^2}$) which is consistent with the WMAP result $n_s = 0.963 \pm 0.012$ [20].

3. GENERATION OF MAGNETIC FIELD DURING HIGGS-INFLATION

We consider the term containing massless vector field separately from the Einstein action,

$$S_E = \int d^4x \sqrt{-\hat{g}} \left[-\frac{1}{4} I^2(\hat{\phi}) F^{\mu\nu} F_{\mu\nu} \right] \quad (18)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $A_\mu = (A_0, A_i)$. Decomposing the spatial part A_i in terms of its transverse and longitudinal components $A_i = A_i^T + \partial_i \chi$ and considering $\partial_i A_i^T = 0$ and $A_0 = \chi'$, we get the action as follows,

$$S_E = \int d^4x I^2 (A_i^{T'} A_i^{T'} + A_i^T \Delta A_i^T) \quad (19)$$

where primes denotes the derivative with respect to the conformal time τ . The transverse component of A_i can written in Fourier space as,

$$A_i^T(x, \hat{\tau}) = \sum_{\sigma=1,2} \int \frac{d^3k}{(2\pi)^{3/2}} A_{\mathbf{k}}^\sigma(\hat{\tau}) \varepsilon_i^\sigma(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \quad (20)$$

where $\varepsilon_i^\sigma(\mathbf{k})$, $\sigma = 1, 2$ are two orthogonal polarization vectors and satisfying the relations $k_i \varepsilon_i^\sigma(\mathbf{k}) = 0$ and $\varepsilon_i^\sigma(-\mathbf{k}) \varepsilon_i^\rho(\mathbf{k}) = \delta^{\sigma\rho}$, the action will be,

$$S_E = \frac{1}{2} \sum_{\sigma=1,2} \int I^2 (A_{\mathbf{k}}^{\sigma'} A_{-\mathbf{k}}^{\sigma'} - k^2 A_{\mathbf{k}}^\sigma A_{-\mathbf{k}}^\sigma) d\hat{\tau} d^3k \quad (21)$$

Defining $A_{\mathbf{k}}^\sigma = \frac{\tilde{A}_{\mathbf{k}}^\sigma}{I}$, the action becomes,

$$S_E = \frac{1}{2} \sum_{\sigma=1,2} \int \left[\tilde{A}_{\mathbf{k}}^{\sigma'} \tilde{A}_{-\mathbf{k}}^{\sigma'} - \left(k^2 - \frac{I''}{I} \right) \tilde{A}_{\mathbf{k}}^\sigma \tilde{A}_{-\mathbf{k}}^\sigma \right] d\hat{\tau} d^3k \quad (22)$$

We can expand $\tilde{A}_{\mathbf{k}}^\sigma$ in terms of the creation and annihilation operators as follows,

$$\tilde{A}_{\mathbf{k}}^\sigma = \frac{1}{\sqrt{2}} \left(u_{\mathbf{k}} a_{\mathbf{k}}^\sigma + u_{\mathbf{k}}^* a_{\mathbf{k}}^{\sigma\dagger} \right) \quad (23)$$

where the creation and annihilation operators satisfies $[a_{\mathbf{k}}^\sigma, a_{\mathbf{k}'}^{\rho\dagger}] = \delta^{\sigma\rho} \delta(\mathbf{k} - \mathbf{k}')$.

Therefore, $u_{\mathbf{k}}$ will satisfy the equation,

$$u_{\mathbf{k}}'' + \left(k^2 - \frac{I''}{I} \right) u_{\mathbf{k}} = 0 \quad (24)$$

For large value of k , equation (24) reduces to

$$u_{\mathbf{k}}'' + k^2 u_{\mathbf{k}} = 0 \quad (25)$$

and the solution will be of the form,

$$u_{\mathbf{k}>} = \frac{1}{\sqrt{2k}} e^{ik\hat{\tau}} \quad (26)$$

But for smaller value of k , the term $\frac{I''}{I}$ will dominate and the solution will be,

$$u_{\mathbf{k}<} = c_1 I + c_2 I \int \frac{d\hat{\tau}}{I^2} \quad (27)$$

Using the relation $\hat{\tau} = -1/\hat{a}\hat{H}$ and the expression of I in equation (14), equation (27) can be simplified to

$$\begin{aligned} u_{\mathbf{k}<} &= \frac{2c_1}{\sqrt{3\xi}} (\log \hat{a})^{1/2} - \frac{c_2}{2\hat{H}} \frac{\sqrt{3\xi}}{\hat{a} (\log \hat{a})^{1/2}} \\ &\simeq \frac{2c_1}{\sqrt{3\xi}} (\log \hat{a})^{1/2} \end{aligned} \quad (28)$$

where second term is neglected as it is suppressed by the factor of $\hat{a} (\log \hat{a})^{1/2}$ in the denominator. By matching the equation (26) and equation (28) at $\hat{\tau} = \frac{1}{k}$, we determine the constant c_1 ,

$$c_1 = \frac{\sqrt{3}e^i}{2} \sqrt{\frac{\xi}{2k \log \hat{a}_k}}$$

where $\hat{a}_k = \frac{k}{\hat{H}}$ is the scale factor at $\hat{\tau} = \frac{1}{k}$. Therefore the solution of the mode functions of the electromagnetic perturbations are of the form,

$$u_{\mathbf{k}} \simeq \frac{e^i}{\sqrt{2k}} \sqrt{\frac{\log \hat{a}}{\log \hat{a}_k}} \quad (29)$$

The correlation function will be,

$$\begin{aligned} \langle 0 | \hat{A}_i^T(\hat{\tau}, x) \hat{A}^{Ti}(\hat{\tau}, y) | 0 \rangle &= \frac{1}{a^2 I^2} \sum_{\sigma\sigma'} \int \frac{d^3 k d^3 k'}{(2\pi)^3} e^{i(\mathbf{k}\cdot\mathbf{x} + \mathbf{k}'\cdot\mathbf{y})} \langle 0 | u_{\mathbf{k}}^\sigma u_{\mathbf{k}'}^{\sigma'} | 0 \rangle \\ &= \frac{1}{4\pi^2 \hat{a}^2 I^2} \int \frac{dk}{k} |u_{\mathbf{k}}|^2 k^3 \frac{\sin k(x-y)}{k(x-y)} \\ &\equiv \int \frac{dk}{k} \delta_A^2(k, \hat{\tau}) \frac{\sin k(x-y)}{k(x-y)} \end{aligned} \quad (30)$$

The power spectrum of the vector field $\delta_A^2(k, \hat{\tau})$ can be identified with,

$$\delta_A^2(k, \hat{\tau}) = \frac{|u_{\mathbf{k}}|^2 k^3}{4\pi^2 \hat{a}^2 I^2} \quad (31)$$

Using the relation between magnetic field and vector field $B^2 = \frac{1}{2a^4} F_{ik} F_{ik} = \frac{1}{a^4} (\partial_i A_k \partial_i A_k - \partial_k A_i \partial_k A_i)$, we can calculate the power spectrum of the magnetic field $\delta_B^2(k, \hat{\tau})$ as follows,

$$\delta_B^2(k, \hat{\tau}) = \delta_A^2(k, \hat{\tau}) \frac{k^2}{\hat{a}^2} = \frac{|u_{\mathbf{k}}|^2 k^5}{4\pi^2 \hat{a}^4 I^2} \quad (32)$$

Using the expression for $u_{\mathbf{k}}$ from equation(29), we can calculate δ_B , at the time of horizon crossing as follows,

$$\delta_B^2(k) \simeq \frac{3}{32\pi^2} \frac{\hat{H}^4 \xi}{|\log \frac{k}{\hat{H}}|} \quad (33)$$

At the time of inflation $\delta_B^2 = 1.9 \times 10^{53} \text{ GeV}^4$ for modes of with a co-moving coherence length $k^{-1} = 100 \text{ kpc}$. After horizon exit δ_B^2 varies as $\frac{1}{\hat{a}^4}$, so we can calculate $\delta_{B_0}^2$ at present. If N is the no of e-foldings after the some specific mode (like $k = 100 \text{ kpc}$), leaves the de-Sitter horizon, then $\delta_{B_0}^2$ and δ_B^2 are related as follows,

$$\delta_{B_0}^2 = \delta_B^2 \left(\frac{\hat{a}_I}{\hat{a}_0} \right)^4 = \delta_{B_I}^2 \exp(-4N) \quad (34)$$

And we find that $N = 61$ gives the value of magnetic perturbation at the present epoch, $\delta_{B_0} = 1.5 \times 10^{-26} \text{ GeV}^2 = 1.7 \times 10^{-6} \text{ Gauss}$ at length scales of 100kpc. We have to study the back reaction of the generated electromagnetic field on the background. For this, we will calculate the energy density ρ_{em} which is defined as T_0^0 component of the energy-momentum tensor.

$$T_0^0 = I^2 \left(\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - F_{0\alpha} F^{0\alpha} \right) = \frac{I^2}{2\hat{a}^4} (A_i^{T'} A_i^{T'} + \partial_i A_k^T \partial_i A_k^T) \quad (35)$$

Using the relation (23), the energy density ρ_{em} will be,

$$\rho_{em} = \langle 0 | \hat{T}_0^0 | 0 \rangle = \frac{1}{8\pi^2 \hat{a}^4} \int \frac{dk}{k} (|u'_{\mathbf{k}}|^2 + k^2 |u_{\mathbf{k}}|^2) k^3 \quad (36)$$

neglecting the derivatives of I . Using the solution (29) for the mode functions expression in equation(36), the energy density perturbations of electromagnetic fields,

$$\begin{aligned} \rho_{em} &= \frac{\hat{H}^4}{64\pi^2} + \frac{\hat{H}^4}{8\pi^2} \left[\frac{1}{16 \log \hat{a}} \left(\frac{1}{(\log \hat{a})^2} + \frac{2}{\log \hat{a}} - \left(\frac{\hat{a}_i}{\hat{a}} \right)^2 \frac{2}{\log \hat{a}_i} - \left(\frac{\hat{a}_i}{\hat{a}} \right)^2 \frac{1}{(\log \hat{a}_i)^2} \right) \right. \\ &\quad \left. + \frac{\log \hat{a}}{32} \left(\frac{1}{(\log \hat{a})^2} - \left(\frac{\hat{a}_i}{\hat{a}} \right)^4 \frac{4}{\log \hat{a}_i} - \left(\frac{\hat{a}_i}{\hat{a}} \right)^4 \frac{1}{(\log \hat{a}_i)^2} \right) \right] \\ &\simeq 8.01 \times 10^{49} \text{ GeV}^4 \end{aligned} \quad (37)$$

We see that the energy density of the electromagnetic fields (37) is smaller than the perturbation of the magnetic field $\delta_B^2 \simeq 10^{53} \text{ GeV}^4$. This is due to the fact that the electromagnetic coupling $g = 1/I$ becomes large (of the order 10) in the Einstein frame during inflation. This means that perturbative calculations in expansion of g are invalid in the regime $\phi > M_P/\xi$ which holds during inflation.

4. CONCLUSION

In this paper we have shown that the Higgs model of inflation [13, 14] in which a large Higgs-Ricci coupling gives rise to a flat Higgs potential in the Einstein frame in early universe, is also ideal for generation of magnetic field during inflation. Breaking the conformal invariance of electromagnetism by a non-renormalizable Higgs-photon coupling term in the Jordan frame enables us to generate large scale magnetic field during inflation while keeping the backreaction pointed out in [9–12] under control. The consequences of primordial magnetic field fluctuations on the CMB anisotropy has been studied in [21, 22]. The cosmological isotropy is broken by large scale magnetic fields which will show up in the CMB anisotropy and polarization spectrum. This points to the possibility that the magnetic field generation model studied in this paper can be tested in the forthcoming CMB anisotropy measurement experiments like PLANCK [23].

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